***NUMBERS AND OPERATIONS***

INTEGRATED GEOMETRY

MS. HUNT

*NAME \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

|  |  |
| --- | --- |
| **M11.A.3.2.1** | Use estimation to solve problems. |
| **M11.A.1.2.1** | Find the Greatest Common Factor (GCF) and/or the Least Common Multiple (LCM) for sets of monomials. |
| **M11.A.1.3.1** | Locate/identify irrational numbers at the approximate location on a number line. |
| **M11.A.1.3.2** | Compare and/or order any real numbers (rational and irrational may be mixed). |
| **M11.A.3.1.1** | Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used). |
| **M11.A.1.1.1** | Find the square root of an integer to the nearest tenth using either a calculator or estimation. |
| **M11.A.1.1.3** | Simplify square roots. (e.g., √24 = 2√6) |
|  |  |
| **M11.A.2.1.1** | Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.). |
| **M11.A.2.1.2** | Solve problems using direct and inverse proportions. |
| **M11.A.2.1.3** | Identify and/or use proportional relationships in problem solving settings. |
|  |  |
| **M11.A.2.2.1** | Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10). |
| **M11.A.2.2.2** | Simplify/evaluate expressions involving multiplying with exponents (e.g. x6 \* x7 = x13), powers of powers (e.g., (x6)7=x42) and powers of products (2x2)3=8x6 (positive exponents only). |
| **M11.A.1.1.2** | Express numbers and/or simplify expressions using scientific notation (including numbers less than 1). |

*M11.A.3.2.1 – Use estimation to solve problems.*

USING ESTIMATION IN REAL-WORLD APPLICATIONS

Directions: Use Estimation to answer the following problems.

* Jeremy had $295.48 in his bank account. He deposited a check for $196.68, withdrew $65.00 and wrote a check for $57.49. What is the ESTIMATED balance of Jeremy’s account after these transactions?
* Henry started a dog-walking service in his neighborhood. He charges $15.00 per hour. He made $611.00 in two weeks. Approximately how many hours did he work in those two weeks.
* Toni went to the comic store. While there he purchased a movie poster for $7.98 and an action figure for $15.50. About how much did Toni spend all together?
* Carolyn rides her bike 2.3 miles from home to school and then 1.82 miles from school to the soccer field. She makes this ride 5 days a week. What is the APPROXIMATE distance that Carolyn rides her bike each week?

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*M11.A.1.2.1 – Find the GCF and/or the LCM for sets of monomials.*

FINDING GCF AND LCM:

Given a pair of positive whole numbers a and b, their greatest common factor (or GCF) is the

largest whole number that divides both a and b.

Given a pair of positive numbers a and b, their least common multiple (or LCM) is the smallest

whole number that is a multiple of both a and b. In other words, the LCM of a and b is the smallest whole

number that is divisible by both a and b.

Try These:

Find the GCF and LCM for each of the sets of numbers:

|  |  |  |  |
| --- | --- | --- | --- |
| a. 20 and 8 | GCF = | | LCM = |
|  | |  | |

|  |  |  |  |
| --- | --- | --- | --- |
| b. 12 and 21 | GCF = | | LCM = |
|  | |  | |

|  |  |  |  |
| --- | --- | --- | --- |
| c. 20 and 36 | GCF = | | LCM = |
|  | |  | |

|  |  |  |  |
| --- | --- | --- | --- |
| d. 12, 18 and 32 | GCF = | | LCM = |
|  | |  | |

*M11.A.1.3.1 – Locating/Identify irrational numbers at the approximate location on a number line.*

*M11.A.1.3.2 – Compare and/or order any real numbers.*

*M11.A.2.2.1 – Evaluate expressions involving absolute value.*

**TYPES OF NUMBERS**

|  |  |  |
| --- | --- | --- |
| **TYPE** | **DEFINITION** | **EXAMPLES** |
| **Natural Numbers**  **or**  **Counting Numbers** |  |  |
| **Whole Numbers** |  |  |
| **Integers** |  |  |
| **Rational Numbers** |  |  |
| **Irrational Numbers** |  |  |
| **Real Numbers** |  |  |

ABSOLUTE VALUE: |a|

The absolute value of a number is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_ that number is from \_\_\_\_\_\_\_\_\_\_\_\_\_\_ on the

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

\* The Absolute Value of a number is NEVER \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

\* Numbers are opposites if their \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is the same.

Try These: Evaluate.

| 10 | = | - 39 | = | t | = | - t | = - | - t | =

| - t | = - | - ( - 4)| = | - 0.18 | = - | - 1.325 | = | ½ | =

Use the number lines below to graph the following sets of numbers:



*\*

*M11.A.3.1.1 – Simplify/evaluate expressions using the order of operations to solve problems (rational #s may be used).*

**Order of Operations** is used to evaluate an expression involving more than one operation.

A helpful way to remember the correct Order of Operations is the statement….

**P**lease **P**arentheses--Grouping Symbols

**E**xcuse **E**xponents

**M**y **D**ear **M**ultiplication / **D**ivision (As it appears **Left to Right**)

**A**unt **S**ally **A**ddition / **S**ubtraction (As it appears **Left to Right**)

Evaluating Expressions with Fraction Bars:

The fraction bar in an expression acts as a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

1. Group everything in the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ together and simplify.
2. Group everything in the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ together and simplify.
3. Then simplify the fraction.

Try These: Simplify.

1) 15 - 2 ⋅ 3 + 1 2) 12 - 32 + 10 ÷ 2 3) 2.4 + 4[ 32 - 2( 3 - 1.5 )]

|  |  |
| --- | --- |
| **#4:** Evaluate:  when *x* = 3 | **#5:** Evaluate:  when *x* = 4 |
| **#6:** Simplify: | **#7:** Simplify: |

|  |  |
| --- | --- |
| **#8:** | **#9:** |

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*M11.A.1.1.1 – Find the square root of an integer to the nearest tenth using either a calculator or estimation.*

*M11.A.1.1.3 – Simplify square roots.*

**SQUARE ROOT OF A NUMBER:** a number that can be multiplied by itself to result in the original

number

\* Squaring a number and Square Rooting a number are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ operations.



**RADICAL SYMBOL**

\*\* Used to represent square roots

**PERFECT SQUARE:** A number whose positive square root is a \_\_\_\_\_\_\_\_\_\_ number

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Perfect Squares** | **# Squared** |  | **Perfect Squares** | **# Squared** |
|  | 22 |  |  | 82 |
|  | 32 |  |  | 92 |
|  | 42 |  |  | 102 |
|  | 52 |  |  | 112 |
|  | 62 |  |  | 122 |
|  | 72 |  |  | 132 |

**Ex:**    

**ESTIMATING SQUARE ROOTS**

- If a number is not a perfect square, you can use perfect squares to estimate the square

root.

**Ex:** 

1. Find two perfect squares that the radicand is between.

 is between  and 

1. Figure out what perfect square is closest to the original number and try squaring

decimals to determine what decimal squared has the closest value to the original radicand.

20 is slightly closer to \_\_\_\_\_, so we will try numbers that are slightly closer to \_\_\_\_\_.



**SIMPLIFYING RADICAL EXPRESSIONS:**

Radicand – The expression UNDER a radical sign.

Square-root expressions with the same radicand are examples of like radicals.

An expression containing a Square-Root is in simplest from when:

* The radicand has no perfect square factors other than 1.
* The radicand has no fractions.
* There are no square roots in any denominator.

|  |  |  |
| --- | --- | --- |
| **PROPERTY** | **EXPLANATION** | **ALGEBRA** |
| Addition/Subtraction Properties | Combine ***like radicals*** by adding or subtracting the numbers multiplied by the radical and keeping the radical the same. |  |
| Product Property | For any non-negative real numbers *a* and *b*, the square root of *ab* is equal to the square root of *a* times the square root of *b*. | Extension : |
| Quotient Property | For any non-negative real numbers *a* and *b* (*a ≥* 0 and *b* > 0), the square root of is equal to the square root of *a* divided by the square root of b. |  |

Examples:

1. B. C.

D. E. F.

G. H. I.

J. K. L.

Rationalizing the Denominator

***Identity Property:***

*A quotient with a square root in the denominator is NOT simplified. Rationalizing the denominator is the process of simplifying a radical expression, by multiplying by a form of 1 to get a perfect-square radicand in the denominator.*

Examples:

M. N.

O. P.

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|  |  |
| --- | --- |
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A Ratio is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of two quantities by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. The ratio *a to b* can also be written: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. A statement that two ratios are equivalent is called a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a ratio of two quantities with different units, such as 34 miles for 2 gallons of gas. Rates are usually written as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. A unit rate is a rate with a second quantity of 1 unit, such as 17 miles per 1 gallon. Other unit rates include, miles per hour, cost per pound, beats per minute and dollars per hour.









**Converting Between Units**

Sometimes we are asked to convert rates to another unit/units. We use ***conversion factors*** to do so. A rate in which the two quantities are equal but use different units, is called a \_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_. Conversion factors include, , , 60 minutes in an hour, 60 seconds in one minute, etc. To convert a rate from one set of units to another, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ by a conversion factor.

* Example: The dwarf sea horse swims at a rate of 52.68 feet per hour. What is the speed in inches per minute?
* A cyclist travels 56 miles in 4 hours. What is the cyclist’s speed in feet per second? Round your answer to the nearest tenth.

**Solving Proportions**

In the proportion , the products and are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

You can solve a proportion for a missing value by using the Cross Products Property, which states that in a proportion the cross products are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Solve each proportion:

A. B. C.

**Using Proportions in Real-World Applications**

A \_\_\_\_\_\_\_\_\_\_\_\_ is a ratio between two sets of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, such as 1 inch:5 miles. A \_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_ or scale model uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing. We can use proportions to find the actual size or distant given the scale for the map or model.

* Example: On the map, the distance from Chicago to Evanston is 0.625 inches. What is the actual distance between the two cities?
* The actual distance between North Chicago and Waukegan is 4 miles. What is this distance on the map? Round to the nearest tenth.
* A scale model of a human heart is 16 feet long. The scale is 32:1. How many inches long is the actual heart it represents?





**Percents**

A percent is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ that compares a number to 100*. For example: 25% = = .*

To find the decimal equivalent of a percent, divide by 100. *For example:*

We can use the proportion to represent percents.

Examples:

|  |  |  |  |
| --- | --- | --- | --- |
| \* | Find 50% of 20. | \* | Find 105% of 72. |
| \* | What percent of 60 is 15? | \* | 440 is what percent of 400? |
| \* | 40% of what number is 14? | \* | 40 is 0.8% of what number? |

*Application – Calculating tip and sales tax are examples of how we use percents on a daily basis.*

* The dinner check for Maria’s family is $67.95. Estimate a 20% tip. Then find the exact 20% tip? A 15% tip?
* The sales tax rate is 6 %. Calculate the sales tax on a shirt that costs $29.50. What is the total cost of the shirt?

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| --- | --- |
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Distributive Property, Properties of Exponents and Scientific Notation

**DISTRIBUTIVE PROPERTY** → *a(b + c) = ab + ac*

The distributive property combines two operations: **Multiplication and Addition.**

Try These: Simplify, combine *like terms* where you can.

1) ( -3x + 7 )11 2) 4( 3 - 5x ) 3) -5( 8 - 4x ) - x 4) 6 + 2( 4x + 3 )

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Exponents

A number written in **Exponential form** has a \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and an \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

**Base -** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

**Exponent - \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** .



*Ex. Identify the base and exponent in Base: \_\_\_\_\_\_ Exponent: \_\_\_\_\_\_*

**PROPERTIES OF EXPONENT**



, if b ≠ 0



, if a ≠ 0



Try These:

In example #1-3, evaluate the expressions with a = 3 and b = -2.

1)  2)  3) 

----------------------------------------------------------------------------------------------------------------------------------Simplify.

4)  5)  6) 

7)  8)  9)  10) 

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SCIENTIFIC NOTATION: *Scientific Notation is a method of writing numbers that are very large or small.*

*A number written in scientific notation has two parts that are multiplied.*

Convert between standard and scientific notation.

|  |  |  |
| --- | --- | --- |
| **Standard Notation** |  | **Scientific Notation** |
| 0.000000000000001 |  |  |
| 40,080,000 |  |  |
|  |  |  |
|  |  |  |

Operations with Scientific Notation: